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1	A^1	B^2	C^3	D^4	E^5	F^6
2	A^1	B^2	D^4	C^3	F^6	E^5
3	A^1	B^2	E^5	F^6	D^4	C^3
4	A^1	B^2	F^6	E^5	C^3	D^4
5	A	C	F	D	B	E
6	A	C	E	B	F	D
7	A	C	B	E	D	F
8	A	D	E	C	B	F
9	A	D	B	F	C	E
10	A	E	D	B	C	F

1	A	B	C	D	E	F	G	H
2	A	B	D	F	G	C	H	E
3	A	B	E	G	F	H	C	D
4	A	B	F	C	H	D	E	G
5	A	B	G	H	C	E	D	F
6	A	B	H	E	D	G	F	C
7	A	C	G	E	H	F	B	D
8	A	C	D	H	E	B	F	G
9	A	C	E	F	B	G	D	H
10	A	C	H	B	F	D	G	E
11	A	C	B	D	G	H	E	F
12	A	D	H	G	E	C	B	F
13	A	D	F	E	G	B	C	H
14	A	D	G	C	B	H	F	E
15	A	D	E	B	C	F	H	G
16	A	E	C	F	D	H	B	G
17	A	E	D	B	H	G	C	F
18	A	E	B	G	C	D	F	H
19	A	F	H	D	B	E	C	G
20	A	F	G	B	D	C	E	H
21	A	G	D	C	F	E	B	H

In seating the 8 persons, the first six arrangements are quite simple, as indicated. Now take 6 reversed, and permute them just as the first 6. This gives the next 5. Take any other, as 3, reversed, and permute in same way. Take 4 in same way rejecting first 1, then 2, then 3, then 4, which have been previously arranged, and we finally get 21.

[Neither of the above solutions show *how* to seat 7 persons in the 15 ways. For the first correct arrangement sent us, we are authorized to offer a year's subscription to the MONTHLY. EDITOR.]

100. Proposed by W. H. CARTER, Vice President, and Professor of Mathemetics, Centenary College, Jackson, La.

$$\text{Solve, } x^{x-y}=y^{4a},\ y^{x+y}=x^{a}.$$

I. Solution by CHARLES C. CROSS, Meredithville, Va.; P. S. BERG, A. M., Larimore, N. D.; J. D. CRAIG, Frankfort, N. J.; JOSIAH H. DRUMMOND, LL. D., Portland, Me.; ALOIS F. KOVARIK, Decorah, Ia.; P. H. PHILBRICK, C. E., Lake Charles, La.; and COOPER D. SCHMITT, A. M., Knoxville, Tenn.

Taking logarithms, we have, $(x+y)\log x=4a\log y\ldots\ldots(1)$;

and $(x+y)\log y=a\log x\ \ldots\ldots(2)$.

(1) divided by (2) gives $\log^2 x=4\log^2 y$,

or $\log x=2\log y\ldots\ldots(3)$;

Therefore $x=y^2 \ldots\ldots(4)$.

Substituting (3) in (2), and dividing by log$y$,

we get $x+y=2a \ldots\ldots(5)$.

Substituting (4) in (5), $y^2+y=2a$.

Therefore, $y=\frac{1}{2}[-1\pm\sqrt{(8a+1)}]$.

$\therefore x=\frac{1}{2}[4a+1\mp\sqrt{(8a+1)}]$.

II. **Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.; J. SCHEFFER, A. M., Hagerstown, Md.; J. W. YOUNG, Fellow and Assistant in Mathematics, Ohio State University, Columbus, O.; ELMER SCHUYLER, B. Sc., Professor of German and Mathematics in Boys' High School, Reading, Pa.**

$$(x+y)\log x=4a\log y \ldots\ldots (1);$$

$$(x+y)\log y=a\log x \ldots\ldots (2).$$

$(1)\div(2)$ gives $(\log x)^2=4(\log y)^2$.

$\therefore \log x=\pm 2\log y$. $\therefore x=y^2$ or $1/y^2$.

$\therefore 2(y^2+y)=4a$. $\therefore y^2+y=2a$.

$\therefore y=-\frac{1}{2}[1\pm\sqrt{(1+8a)}]$; $x=\frac{1}{2}[1+4a\mp\sqrt{(1+8a)}]$.

When $x=1/y^2$, we get $y^3+2ay^2=-1$.

The roots of this equation can be found by Cardan's Rule.

[ZERR, SCHEFFER]

Rational values are obtained by substituting for a the triangular numbers 1, 3, 6, 10, 15, 21, 28, etc. The equation $y^{-2}+y+2a=0$ leads to the cubic $y^3+2ay^2+1=0$. In case of $a=3$, we have the simultaneous sets

$$y=2\ ;\ y=-3\ ;\ y=-6.0275\ ;$$

$$x=4\ ;\ x=9\ ;\ x=-.0275\ ;$$

and two imaginary roots. [SCHEFFER.]

III. **Solution by WALTER HUGH DRANE, Graduate Student, Harvard University, Cambridge, Mass.; and CHAS. E. MYERS, Canton, Ohio.**

$x^{x+y}=y^{4a} \ldots\ldots (1)$. $y^{x+y}=x^a \ldots\ldots(2)$.

From (1) $y=x^{(x+y)/4a}$, and from (2) $y=x^{a/(x+y)}$.

$\therefore x^{(x+y)/4a}=x^{a/(x+y)} \ldots\ldots(3)$. $\therefore (x+y)/4a=a/(x+y) \ldots\ldots (4)$.

$\therefore x+y=\pm 2a \ldots\ldots(5)$.

This in (1) gives, taking + value of a, $x^{2a}=y^{4a}$.

$\therefore x=y^2$. $\therefore$ By (2) $y^{2+y}=y^{2a}$. $\therefore y^2+y=2a$.

$$\therefore y=\frac{-1\pm\sqrt{(1+8a)}}{2};\ \therefore x=\frac{1+4a\mp\sqrt{(1+8a)}}{2}.$$

[DRANE, MYERS.]

Taking the negative value of $2a$ leads us to the cubic $y^3+2ay^2+1=0$, and $x=1/y^2$. Substitute $z=y+2a/3$, and this reduces to $z^3-4a^2z+1=0$. Solving this by Cardan's method gives

$$u=\tfrac{1}{2}[-1\pm\sqrt{(1-\tfrac{256}{27}a^6)}],\text{ and } v=\tfrac{1}{2}[-1\mp\sqrt{(1-\tfrac{256}{27}a^6)}],$$

where $u^{\frac{1}{3}}+v^{\frac{1}{3}}=z$, and $u^{\frac{1}{3}}v^{\frac{1}{3}}=4a^2/3$. $\therefore$ Three values of z are,

$$z_1=\left(\frac{-1\pm\sqrt{(1-\frac{256}{27}a^6)}}{2}\right)^{\frac{1}{3}}+\frac{4a^2\sqrt[3]{2}}{[-1\pm\sqrt{(1-\frac{256}{27}a^6)}]^{\frac{1}{3}}};$$

$$z_2=w\left(\frac{-1\pm\sqrt{(1-\frac{256}{27}a^6)}}{2}\right)^{\frac{1}{3}}+\frac{4a^2\sqrt[3]{2}}{w[-1\pm\sqrt{(1-\frac{256}{27}a^6)}]^{\frac{1}{3}}};$$

$$z_3=w^2\left(\frac{-1\pm\sqrt{(1-\frac{256}{27}a^6)}}{2}\right)^{\frac{1}{3}}+\frac{4a^2\sqrt[3]{2}}{w^2[-1\pm\sqrt{(1-\frac{256}{27}a^6)}]^{\frac{1}{3}}}.$$

$\therefore$ $y_1=z_1-2a/3$, $y_2=z_2-2a/3$, and $y_3=z_3-2a/3$, are the three required values of y; and $x_1=1/y_1^2$, $x_2=1/y_2^2$, $x_3=1/y_3^2$, are the three values of x. It is easily seen that the values of y, and hence of x, are all real, one real and two imaginary, or real and multiple, according as $a^6>$, $<$, or $=\frac{27}{256}$. [DRANE.]

GEOMETRY.

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122. Proposed by G. I. HOPKINS, A. M., Professor of Mathematics and Physics, Manchester High School, Manchester, N. H.

If perpendiculars are dropped from the vertices of a regular polygon upon any diameter of the circumscribed circle, the sum of the perpendiculars which fall on one side of this diameter is equal to the sum of those which fall on the opposite side. [From Chauvenet's *Treatise on Elementary Geometry.*]

III. Solution by WILLIAM H. ECHOLS, B. S., C. E., Professor of Mathematics in University of Virginia, Charlottesville, Va.

A very simple proof of this is:

1°. Let there be a regular polygon of $2n$ sides, $p_1, \ldots\ldots p_{2n}$, the perpendiculars from vertices on any straight line.

Then if p_c is perpendicular from the center, clearly

$$\sum_1^{2n} p_r=2np_c.$$

2°. Form a polygon of n sides by joining the mid-points of alternate sides of 1°. Then at once

$$2\sum_1^{n} p'_r=\sum_1^{2n} p_r=2np_c,$$

where p'_r are perpendiculars of n-side.

This is true whatever integer be n.

3°. If the line of projection passes through the center, the theorem is demonstrated generally.

125. Proposed by J. SCHEFFER, A. M., Hagerstown, Md.

To find the locus of a point on the surface of an ellipsoid which has the property that the tangent plane at that point is at the given distance, f, from the center of the ellipsoid.